

Forecasting KRW-USD Exchange Rate Volatility and Analyzing the Risk Premium

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Abstract

Recently, due to heightened uncertainty surrounding U.S. monetary policy, the volatility of the KRW-USD exchange rate has increased significantly. Therefore, this study transforms the KRW-USD exchange rate data into log return data to forecast exchange rate volatility and analyze the risk premium. The models used to predict volatility are the AR-GARCH model and the AR-GARCH-M model. The log return data were found to follow an AR(2)-GARCH(1,1) process under the AR-GARCH model, and an AR(1)-GARCH(1,1)-M process under the GARCH(p,q)-M model. The goodness-of-fit for the models was tested using the Portmanteau Q-test for autocorrelation, and the results indicated that the models were appropriate. Based on these fitted models, the predicted exchange rate volatility is expected to remain relatively low over a certain period. Furthermore, analysis of the risk premium using the AR(1)-GARCH(1,1)-M model showed that there is no risk premium present in exchange rate returns.

Keywords: *Log Return, Volatility, AR-GARCH Model, AR-GARCH-M Model, Risk Premium.*

Introduction

Since experiencing the 1997 Asian financial crisis and the 2008 global financial crisis, South Korea has made efforts to stabilize its macroeconomy through policy measures and an increase in foreign exchange reserves, which have generally contributed to a reduction in KRW-USD exchange rate volatility. However, more recently, a variety of external and internal factors have led to a renewed increase in exchange rate volatility. External factors include the global economic slowdown, uncertainties in U.S. monetary policy, and geopolitical risks in the Middle East. Internal factors include the possibility of increased capital flow volatility, slowing domestic economic growth, interest rate fluctuations, inflation, and political uncertainty. Rising exchange rate volatility affects various aspects of the real economy, including imports and exports, trade balance, inflation, interest rates, financial markets, and corporate management.

An increase in the exchange rate can enhance the price competitiveness of domestic products, leading to higher exports and lower imports. This, in turn, improves the trade balance and has a positive impact on GDP. In particular, since South Korea is a highly trade-dependent country, a higher exchange rate tends to improve the profitability of export-oriented firms, thereby supporting corporate growth and investment. However, the negative impacts of a rising exchange rate may outweigh the positives in the case of the Korean economy. South Korea relies heavily on imports of raw materials, and fluctuations in global commodity prices, including oil, directly affect domestic inflation. In this context, a rising exchange rate increases the cost of imported raw materials, raising corporate expenses and deteriorating profitability. In addition, higher commodity prices reduce household consumption, leading to a slowdown in overall economic growth. The increase in the exchange rate also adversely affects interest rates and the financial markets. Persistent currency depreciation can raise import prices, increasing inflationary pressure and putting downward pressure on the economy. A sharp depreciation of the won may increase expectations of interest rate hikes, leading to capital outflows by foreign investors, greater bond market volatility, and broader financial instability, including effects on the real estate market. Furthermore, exchange rate hikes reduce corporate profitability, increase stock market volatility, dampen investor sentiment, and heighten uncertainty in financial markets. The exchange rate, as a key macro-financial variable, interacts in complex and multifaceted ways with the broader economy. Therefore, in the current environment where both external and internal factors are raising

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concerns about increased exchange rate volatility, understanding and analyzing this volatility is essential for economic agents engaged in financial transactions.

Literature Review

Volatility estimation involves modeling the heteroskedasticity observed in macroeconomic variables. Volatility tends to change in response to shocks and often takes time to subside after a surge. To estimate the volatility of macroeconomic variables, a variety of GARCH family models have been developed based on the original ARCH model. These GARCH-type models are known for effectively capturing heteroskedasticity and are widely used in estimating the volatility of various economic variables. Previous studies that applied GARCH family models include the following:

Elyasiani and Mansur (1998) used the GARCH-M model to analyze the sensitivity of bank stock return distributions to changes in interest rate levels and volatility. They found that both the interest rate and its volatility have a direct impact on the first and second moments of bank stock returns [1]. Tai (2000), using a nonlinear SUR model and a multivariate GARCH-M model, estimated market risk, interest rate risk, and exchange rate risk, concluding that interest rate and exchange rate risks vary over time [2]. Elyasiani and Mansur (2004) found that short- and long-term interest rates and their volatilities have statistically significant and distinct impacts on the return-generating processes of bank portfolios, using a multivariate GARCH-M model [3].

Engle and Rangel (2008) applied the spline-GARCH model to estimate the volatility of macroeconomic variables in 50 countries, showing that countries with larger economic zones tend to exhibit greater volatility. They also argued that their proposed model is suitable for long-term forecasting [4]. Chauvet et al. (2012) demonstrated that stock volatility at the industry level and bond market volatility, derived from daily returns, are useful for predicting real economic activity [5]. Asgharian, Hou, and Javed (2013) extracted common factors from macroeconomic variables using principal component analysis and applied the GARCH-MIDAS model, finding that incorporating macroeconomic information enhances both forecasting accuracy and long-term variance predictions [6]. Conrad and Loch (2015) used the GARCH-MIDAS model to analyze the relationship between stock market risk and macroeconomic conditions, confirming its strong predictive power for stock market volatility [7]. Seung Hee Lee and Hee Joon Han (2016) analyzed the volatility of KOSPI index returns using a GARCH model and a single-indicator volatility model that captures long-term variation. They found that the housing price index represents the long-term volatility in the Korean stock market [8]. Young Im Lee and Jin Lee (2017) applied the GARCH-MIDAS model while considering domestic and international economic variables and concluded that foreign economic variables effectively explain volatility in the Korean stock market [9]. Do Kyun Chun (2017) estimated and compared the volatilities of the KRW-USD, KRW-JPY, KRW-EUR, and KRW-GBP exchange rates using both GARCH and stochastic volatility models [10]. Conrad and Kleen (2020) applied the mixed-frequency GARCH model to macroeconomic return data and confirmed that the model effectively explains autocorrelation patterns in return data [11]. Elder and Payne (2023) applied a multivariate GARCH-in-Mean model to analyze the impact of oil price uncertainty on unemployment rates across different racial and ethnic groups, finding that oil price uncertainty shocks have a greater impact on male unemployment rate volatility compared to that of females [12].

As these previous studies show, GARCH-type models are useful tools for estimating and forecasting the volatility of macroeconomic and financial variables. Accordingly, this study employs the AR-GARCH and AR-GARCH-M models to forecast the volatility of the KRW-USD exchange rate and to analyze the associated risk premium.

Research Model

Data Characteristics

The data used in this study consists of monthly average KRW-USD exchange rate figures from January 2001 to December 2024, obtained from the Economic Statistics System of the Bank of Korea. To estimate volatility and examine the presence of a risk premium, the exchange rate data was transformed into log return form. Let P_t denote the KRW-USD exchange rate at time t , then the log return of the exchange rate is represented by $Z_t = \log(P_t/P_{t-1})$.

AR(p)-GARCH(p,q) Model

To model the error term, this study adopts the AR(p)-GARCH(p,q) model, which combines an autoregressive model of order p for the mean equation with a GARCH(p,q) model for the conditional

variance of the error term. The model is defined as follows (Equation 1):

$$\begin{aligned}
 Z_t &= x_t' \beta + \varepsilon_t \\
 \varepsilon_t &= \phi_1 \varepsilon_{t-1} + \cdots + \phi_p \varepsilon_{t-p} + \nu_t \\
 \nu_t &= a_t \sigma_t, \quad a_t \sim i.i.d. N(0,1) \\
 \sigma_t^2 &= \alpha_0 + \sum_{i=1}^q \alpha_i \nu_{t-i} + \cdots + \sum_{j=1}^p \delta_j \sigma_{t-j}^2
 \end{aligned} \tag{Equation 1}$$

In Equation (1), for the model to be estimated stably, it must satisfy the non-negativity condition $p \geq 0$, $q > 0$, $\alpha_0 > 0$, $\alpha_i \geq 0$, and the stationarity condition $\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \delta_j < 1$.

AR(p)-GARCH(p,q)-M Model

To analyze the risk premium associated with exchange rate volatility, this study uses the GARCH-in-Mean (GARCH-M) model, in which mean of the log return Z_t depends on its conditional variance σ_t^2 . The model is specified as follows (Equation 2):

$$\begin{aligned}
 Z_t &= x_t' \beta + c h(\sigma_t) + \varepsilon_t \\
 \varepsilon_t &= \phi_1 \varepsilon_{t-1} + \cdots + \phi_p \varepsilon_{t-p} + \nu_t \\
 \nu_t &= a_t \sigma_t, \quad a_t \sim i.i.d. N(0,1) \\
 \sigma_t^2 &= \alpha_0 + \sum_{i=1}^q \alpha_i \nu_{t-i} + \cdots + \sum_{j=1}^p \delta_j \sigma_{t-j}^2
 \end{aligned} \tag{Equation 2}$$

In Equation (2), c represents the risk premium parameter, and $c h(\sigma_t)$ tests the risk premium in the same way as in (Equation 3):

$$\begin{aligned}
 Z_t &= x_t' \beta + c h(t) + \varepsilon_t \\
 Z_t &= x_t' \beta + c \log(h_t) + \varepsilon_t \\
 Z_t &= x_t' \beta + c \sqrt{h_t} + \varepsilon_t
 \end{aligned} \tag{Equation 3}$$

According to the test results, if $c > 0$, a risk premium exists. If $c = 0$, it implies that no risk premium is present.

Research results Characteristics of Log Return Data

The log return data for the KRW-USD exchange rate, as shown in Figure 1, is identified as a stationary time series with no special patterns such as non-stationarity or trends.

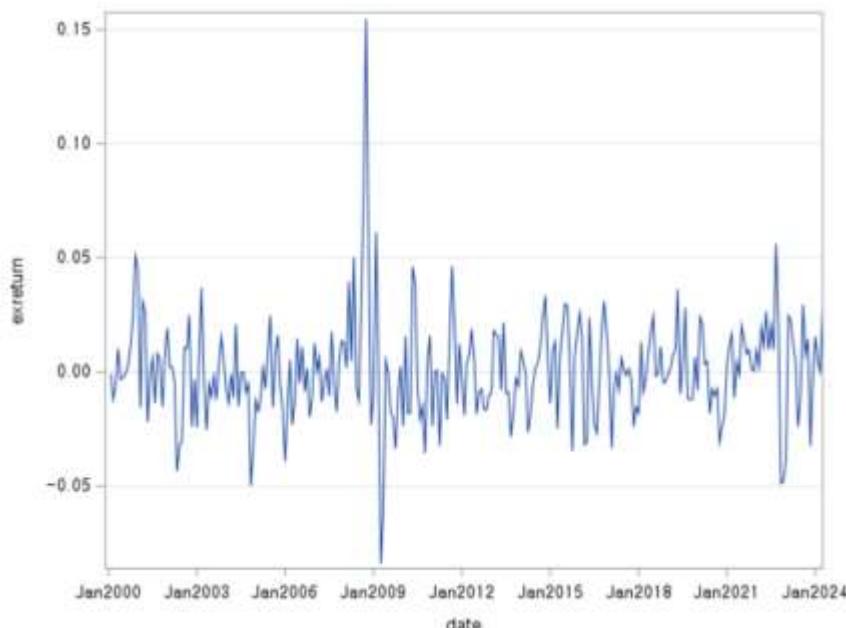


Fig. 1. Log Return Time Series Plot

However, when testing for autocorrelation using the Portmanteau Q-test, the log return series of the KRW-USD exchange rate showed significant autocorrelation at all lags Table 1. This indicates that the log return data is not independently distributed and that autocorrelation exists in the variance(σ^2). In this study, we present the test results for all lags up to 36 at a 5% significance level.

Table 1. Autocorrelation Test for Log Returns

To Lags	Chi-Square	Pr > ChiSq	Autocorrelations					
6	46.53	<.0001	0.369	-0.049	-0.061	0.036	0.100	0.022
12	56.30	<.0001	-0.118	-0.081	0.056	0.048	-0.014	-0.073
18	65.64	<.0001	-0.140	-0.035	0.031	-0.028	-0.081	-0.021
24	77.41	<.0001	0.072	0.036	0.043	-0.085	-0.133	-0.053
30	79.32	<.0001	0.008	0.014	0.003	-0.059	-0.043	-0.014
36	87.54	<.0001	0.020	-0.062	-0.067	0.008	0.100	0.074

Specification of the Error Term Autoregressive Model

To determine the appropriate autoregressive structure of the error term, we conducted autocorrelation tests using the Minimum Information Criterion (MINIC) proposed by Hannan and Rissanen, and the method suggested by Said and Dickey. As a result, the error term was found to follow an AR(p) process with p=6. Accordingly, the model included autocorrelation up to lag 6, and through backward elimination, significant parameters estimated were ϕ_1 and ϕ_2 , as shown in Table 2.

Table 2. Parameter Estimates for AR(6) and AR(2)

AR (6) Model Parameter Estimates				
Variable	Estimate	S. E	t -Value	Pr > t
Intercept	0.000824	0.001727	0.49	0.6263
AR1	-0.4573	0.0586	-7.80	<.0001
AR2	0.2289	0.0643	3.56	0.0004

AR3	-0.0501	0.0659	-0.76	0.4476
AR4	0.002357	0.0659	0.04	0.9715
AR5	-0.0885	0.0646	-1.37	0.1716
AR6	0.0439	0.0589	0.74	0.4569
AR (2) Model Parameter Estimates				
Intercept	0.000824	0.001532	0.54	0.5912
AR1	-0.4483	0.0569	-7.88	<.0001
AR2	0.2139	0.0570	3.76	0.0002

The Portmanteau Q-test applied to the AR(2) error model indicated that the p-values of the chi-square statistic were very small across all lags. Thus, the AR(2) model is identified as a suitable model with no autocorrelation in the residuals Table 3.

Table 3. Autocorrelation Test for AR(2) Error Model

To Lags	Chi-Square	Pr > ChiSq	Autocorrelations					
6	2.36	0.8841	0.013	-0.009	0.040	0.020	0.065	0.035
12	8.64	0.7337	-0.098	-0.065	0.075	0.004	-0.027	-0.004
18	16.42	0.5630	-0.135	-0.011	0.035	-0.026	-0.061	-0.025
24	26.70	0.3185	0.088	-0.048	0.082	-0.074	-0.095	-0.019
30	28.70	0.5335	0.004	-0.018	0.028	-0.059	-0.014	-0.036
36	34.31	0.5493	0.051	-0.062	-0.036	-0.005	0.075	0.056

Fitting the Conditional Heteroskedasticity Model

In order to verify the presence of volatility in the AR(2) error term model, a Portmanteau Q-test was performed on the squared residuals obtained from the AR(2) error term model, and the results showed that volatility existed in all lags Table 4.

Table 4. Autocorrelation Test for Volatility in AR(2) Error Model

To Lags	Chi-Square	Pr > ChiSq	Autocorrelations					
6	48.55	<.0001	0.163	0.099	-0.009	0.185	0.161	0.249
12	52.64	<.0001	0.101	-0.015	0.004	-0.030	0.044	-0.009
18	52.94	<.0001	0.023	-0.011	-0.005	-0.011	0.007	-0.011
24	55.79	0.0002	0.076	-0.000	-0.017	-0.039	-0.008	0.035
30	56.57	0.0023	-0.005	-0.029	-0.004	-0.019	-0.017	0.028
36	57.41	0.0131	-0.020	-0.029	0.019	-0.015	0.006	-0.025

Therefore, to fit a GARCH model to the AR(2) error model, the maximum likelihood estimation (MLE) method was applied. Starting from a GARCH(1,6) model, insignificant parameters were excluded iteratively. The final results showed that all parameter estimates in the AR(2)-GARCH(1,1) model were statistically significant Table 5.

Table 5. Parameter Test for AR(2)-GARCH(1,1) Model

Variable	Estimate	S. E	t -Value	Pr > t
Intercept	-0.000237	0.001451	-0.16	0.8701
AR1	-0.3746	0.0737	-5.08	<.0001

AR2	0.1406	0.0682	2.06	0.0394
ARCH0	0.0000571	0.0000291	1.96	0.0498
ARCH1	0.1623	0.0435	3.73	0.0002
GARCH1	0.6902	0.1109	6.23	<.0001

A subsequent autocorrelation test on the squared residuals of the AR(2)-GARCH(1,1) model showed no significant autocorrelation at all lags, indicating a well-fitted model Table 6.

Table 6. Autocorrelation Test for AR(2)-GARCH(1,1) Model

Residual squared									
To Lags	Chi-Square	Pr > ChiSq	Autocorrelations						
6	7.46	0.2802	-0.002	0.057	-0.052	0.030	0.083	0.103	
12	15.33	0.2237	0.045	-0.062	-0.018	-0.110	0.018	-0.081	
18	18.50	0.4232	-0.006	0.052	-0.080	-0.014	0.025	-0.007	
24	23.48	0.4915	0.077	-0.007	-0.045	-0.068	0.030	0.044	
30	24.86	0.7318	0.019	-0.022	-0.016	-0.024	-0.030	0.040	
36	26.31	0.8819	0.002	-0.038	-0.040	-0.030	-0.014	-0.010	

Volatility Forecasting Using the AR(2)-GARCH(1,1) Model

The 1-lag predicted values of volatility during the fitting period and the multi-lag predicted values of volatility during the forecast period using the AR(2)-GARCH(1,1) model are as shown in Figure 2, and it is predicted that volatility will not fluctuate significantly during a certain period.

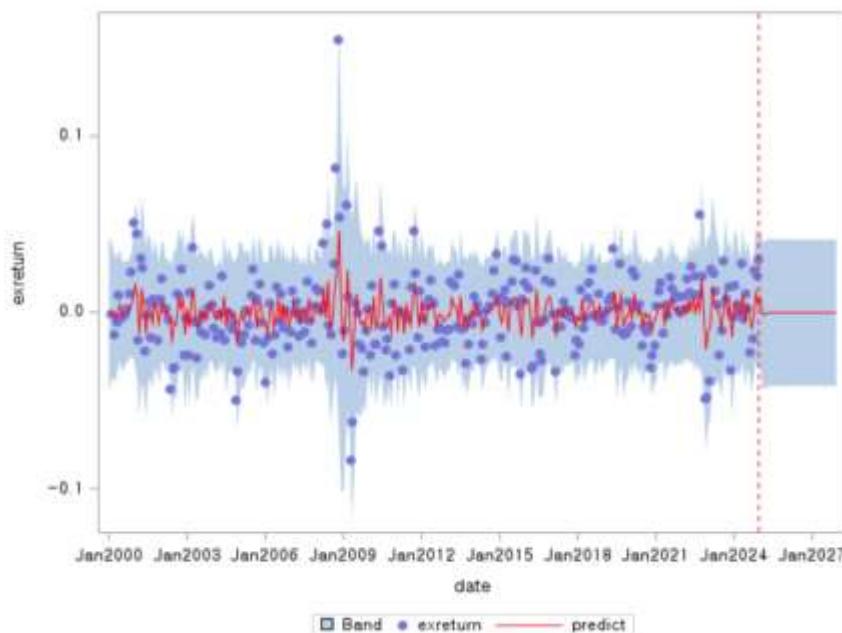


Fig. 2. Volatility Forecast from AR(2)-GARCH(1,1) Model

Risk Premium Analysis

When fitting a GARCH-M model to the AR(2)-GARCH(1,1) structure, the error term was found to follow an AR(1) process. Thus, the AR(1)-GARCH(1,1)-M model was estimated using MLE. All parameter estimates were found to be significant Table 7, and the squared residuals showed no autocorrelation at all lags, confirming model adequacy Table 8.

Table 7. Parameter Test for AR(1)-GARCH(1,1)-M Model

Variable	Estimate	S. E	t -Value	Pr > t
Intercept	0.002584	0.006671	0.39	0.6984
AR1	-0.3332	0.0771	4.32	<.0001
ARCH0	0.0000223	9.1299E-6	2.44	0.0146
ARCH1	0.2250	0.0523	4.30	<.0001
GARCH1	0.7750	0.0523	14.82	<.0001

Table 8. Autocorrelation Test for AR(1)-GARCH(1,1)-M Model

Residual squared								
To Lags	Chi-Square	Pr > ChiSq	Autocorrelations					
6	4.33	0.6315	-0.030	0.048	-0.059	-0.009	0.022	0.084
12	11.70	0.4701	0.020	-0.075	-0.016	-0.104	-0.018	-0.078
18	17.01	0.5221	-0.034	0.059	-0.098	-0.033	0.037	0.009
24	20.95	0.6419	0.028	-0.015	-0.046	-0.086	0.039	0.002
30	22.43	0.8381	0.012	-0.019	-0.004	-0.034	-0.039	0.036
36	23.94	0.9382	0.006	-0.046	0.035	-0.021	-0.026	0.004

To examine the relationship between exchange rate volatility and returns, the AR(1)-GARCH(1,1)-M model was used to test for the presence of a risk premium. The test results indicated that no risk premium exists in the return on the exchange rate Table 9.

Table 9. Risk Premium Analysis

Model	Estimate	S. E	t -Value	Pr > t
$Z_t = x_t' \beta + c h_t + \varepsilon_t$	-1.7008	7.5259	-0.23	0.8212
$Z_t = x_t' \beta + c \log(h_t) + \varepsilon_t$	-0.003034	0.003706	-0.82	0.4130
$Z_t = x_t' \beta + c \sqrt{h_t} + \varepsilon_t$	-0.1550	0.3670	-0.42	0.6729

Forecasting with the AR(1)-GARCH(1,1)-M Model

Using the AR(1)-GARCH(1,1)-M model, one-step-ahead and multi-step-ahead volatility forecasts were generated. As shown in Figure 3, the forecasts are similar to those from the AR(2)-GARCH(1,1) model and suggest that volatility will remain stable over a certain period.

The 1-lag predicted values of volatility during the fitted period and the multi-lag predicted values of volatility during the forecast period are as shown in Figure 3 using the AR(1)-GARCH(1,1)-M model. Similar to the volatility prediction of the AR(1)-GARCH(1,1) model, it is predicted that volatility will not fluctuate significantly during a certain period.

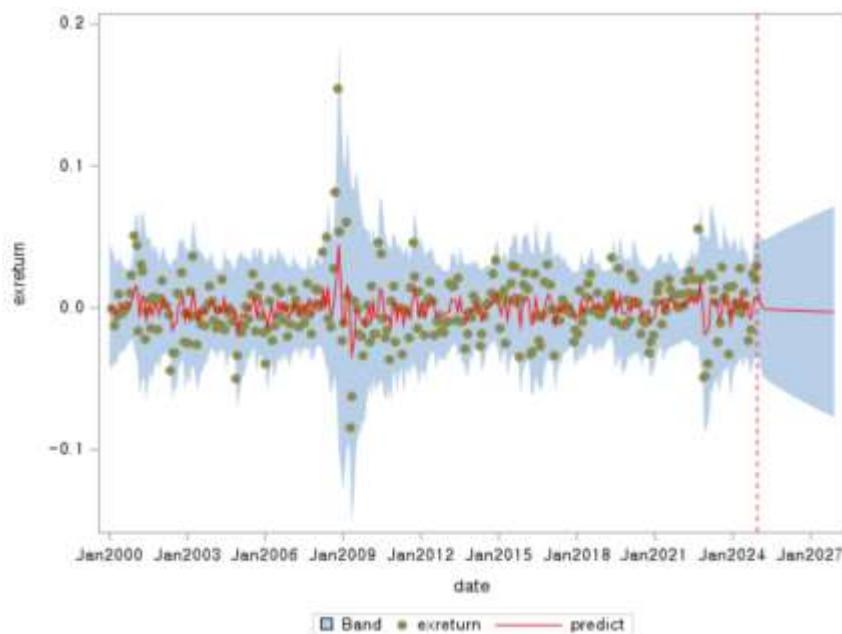


Fig. 3. Volatility Forecasting by AR (3,6)-GARCH (1,1) Model

Conclusion

Concerns about the recent strengthening of the U.S. dollar are growing. This dollar appreciation poses a significant risk to countries with high external dependence, such as South Korea, both economically and financially. A strong dollar increases global economic uncertainty and, in turn, amplifies exchange rate volatility. Given the ongoing uncertainty surrounding U.S. monetary policy and the global economy, the findings of this study can be summarized as follows.

The error terms in the log return series of the KRW-USD exchange rate were found to follow an autoregressive process based on autocorrelation test results. The lag order of the autoregressive model was determined using backward elimination, and volatility was confirmed by the residual square obtained after fitting the error term autoregressive model. Therefore, in order to fit the GARCH model to the error term model, insignificant parameters were excluded and the re-estimation process was repeated to establish the AR-GARCH model. Furthermore, when the AR-GARCH-M model was applied, it was found that the only change was a reduction in the lag of the error term by one. After fitting both the AR-GARCH and AR-GARCH-M models, the parameter estimates were found to be statistically significant, and the autocorrelation tests using squared residuals confirmed that there was no remaining autocorrelation. Volatility forecasts from both models indicated that exchange rate volatility is expected to remain relatively low over the short term. Additionally, the risk premium analysis using the AR-GARCH-M model showed that there is no significant risk premium in the exchange rate returns.

Although it is difficult to predict how long the strong dollar will persist, the findings of this study suggest the importance of analyzing the current economic conditions based on empirical evidence and preparing appropriate policy responses if necessary. For the sustainable growth of the Korean economy, it is essential to closely monitor changes in the global economy and establish strategic measures for managing foreign exchange market risks and responding to economic uncertainties.

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